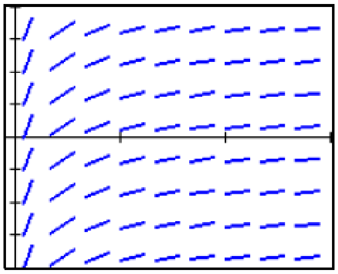
AP Calculus AB

Unit 11: Differential Equations

AP Calculus AB - Worksheet 94 **Slope Fields**

The slope field from a certain differential equation is shown for each problem. For each, identify either the differential equation OR particular solution that is associated with that slope field.

1.



(A) $y = \ln x$

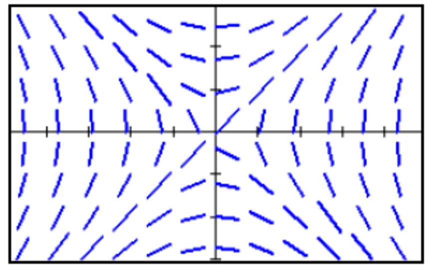
(B) $y = e^x$

(C) $y = e^{-x}$

(D) $y = \cos x$

(E) $y = x^2$

2.



(A) $\frac{dy}{dx} = x + y$

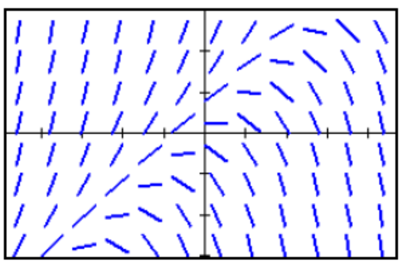
(B) $\frac{dy}{dx} = \frac{x}{y}$

(C) $\frac{dy}{dx} = \frac{y}{x}$

(D) $\frac{dy}{dx} = (x-1)y$

(E) $\frac{dy}{dx} = x(y-1)$

3.



(A) $\frac{dy}{dx} = y - x$

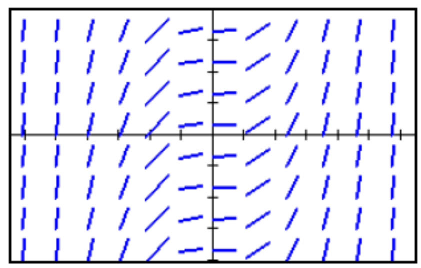
(B) $\frac{dy}{dx} = -\frac{x}{y}$

(C) $\frac{dy}{dx} = -\frac{y}{x}$

(D) $\frac{dy}{dx} = y(x-1)$

(E) $\frac{dy}{dx} = x(y-1)$

4.



(A) $y = \sin x$

(B) $y = \cos x$

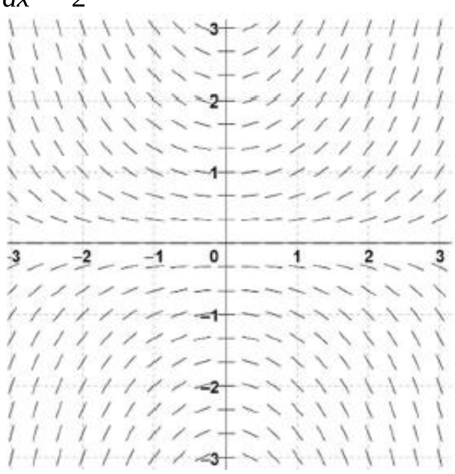
(C) $y = x^2$

(D) $y = \frac{1}{6}x^3$

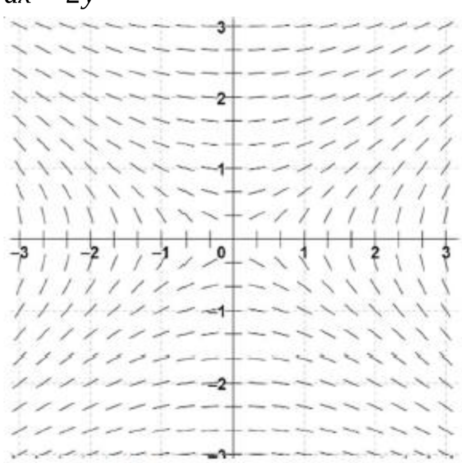
(E) $y = \frac{1}{4}x^4$

For each slope field, plot and label the points A and B and sketch the particular solution that passes through each of those points. (Two solutions for each slope field.)

5. $\frac{dy}{dx} = \frac{xy}{2}$; Point A: $(0,1)$; Point B: $(-2,-1)$



6. $\frac{dy}{dx} = \frac{x}{2y}$; Point A: $(0,1)$; Point B: $(-2,-1)$

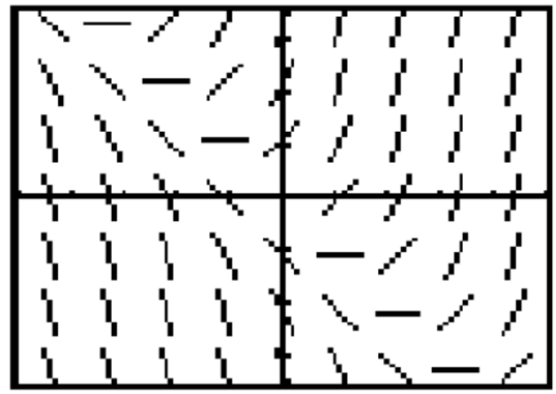


7. The slope field for the differential equation $\frac{dy}{dx} = x + y$ is shown in the figure to the right.

a) Sketch the solution curve through the point $(0,1)$.

b) Sketch the solution curve through the point $(-3,0)$.

c) Use the tangent line to the curve $y = f(x)$ at the point $(-3,0)$ to approximate $y(-3.1)$

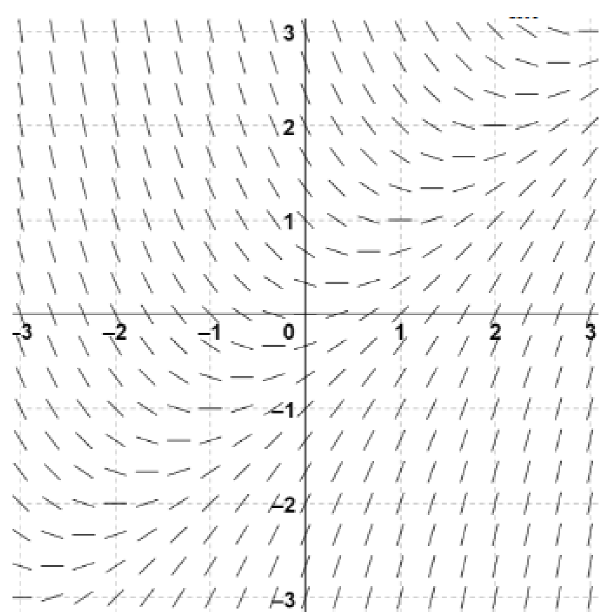


8. The slope field for the differential equation $\frac{dy}{dx} = x - y$ is shown in the figure to the right.

a) Sketch the graph of the particular solution that contains $(-1,-1)$.

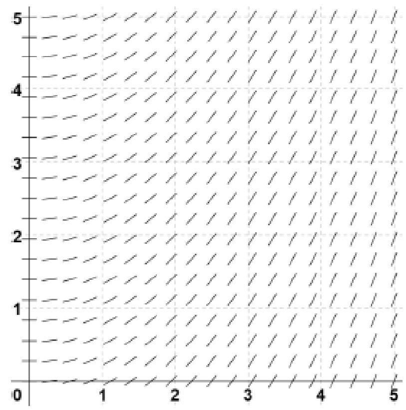
b) Sketch the graph of the particular solution that contains $(1,-1)$.

c) State a point, other than the origin, where $\frac{dy}{dx} = 0$. Find $\frac{d^2y}{dx^2}$ and use it to verify if your point is a local max or min.

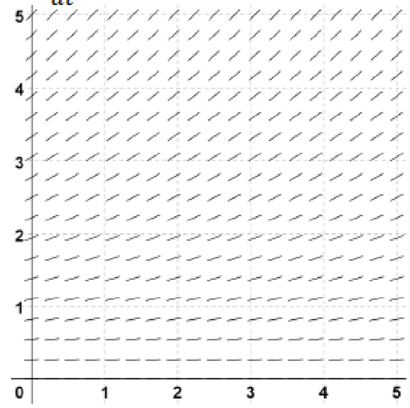


For each problem below a slope field and a differential equation are given. Explain why the slope field CANNOT represent the differential equation.

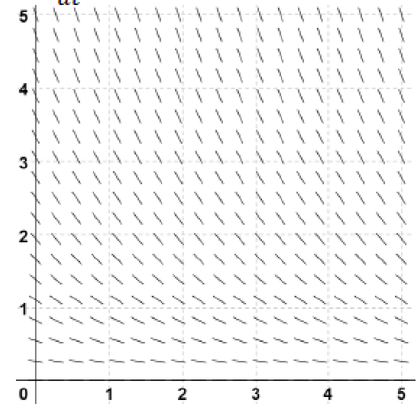
9. $\frac{dy}{dt} = 0.5y$



10. $\frac{dy}{dt} = -0.2y$

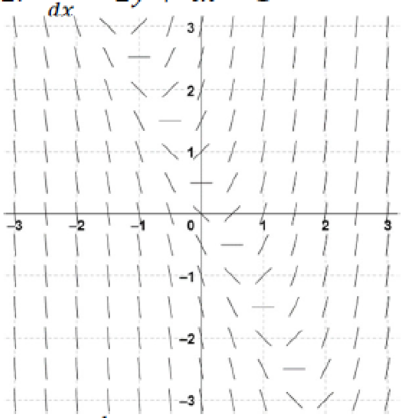


11. $\frac{dy}{dt} = 0.6y$



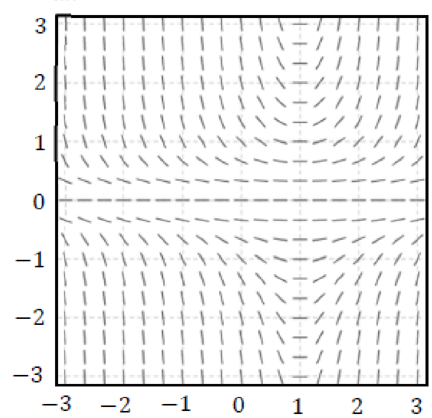
Consider the differential equation and its slope field. Describe all points in the xy-plane that match the given condition.

12. $\frac{dy}{dx} = 2y + 4x - 1$



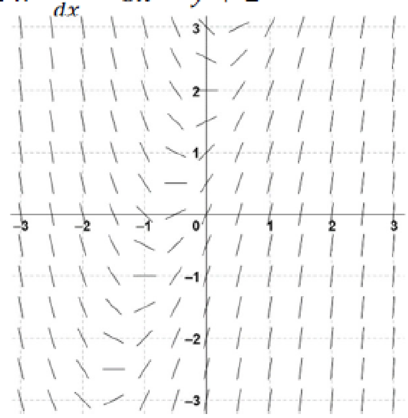
When is $\frac{dy}{dx}$ positive?

13. $\frac{dy}{dx} = y^2(x - 1)$



When are the slopes nonnegative?

14. $\frac{dy}{dx} = 3x - y + 2$



When does $\frac{dy}{dx} = 1$?

AP Calculus AB - Worksheet 95 **Solving Differential Equations – Separation of Variables**

Solve each differential equation by using separation of variables.

1.	$\frac{dy}{dx} = xy^2$
2.	$\frac{dy}{dx} = 9y$
3.	$\frac{dy}{dx} = y\cos x$
4.	$\frac{dy}{dx} = 2\sqrt{x}$

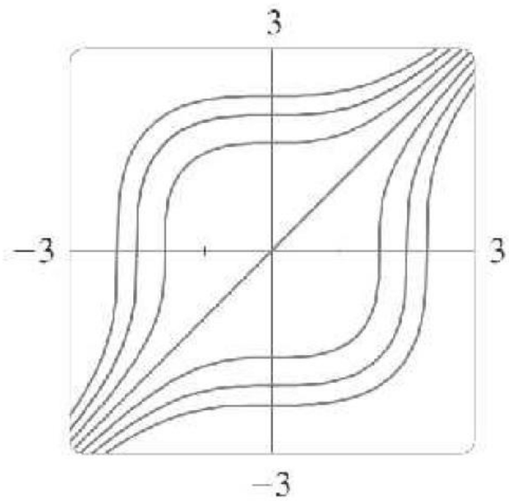
Use separation of variables to find the solution to the initial value problem. Indicate the domain over which the solution is valid

5.	$\frac{dy}{dx} = \frac{x}{y}$ and $y = 2$ when $x = 1$
6.	$\frac{dy}{dx} = -\frac{x}{y}$ and $y = 3$ when $x = 4$
7.	$\frac{dy}{dx} = \frac{y}{x}$ and $y = 2$ when $x = 2$
8.	$\frac{dy}{dx} = 2xy$ and $y = 3$ when $x = 0$
9.	$\frac{dy}{dx} = (y+5)(x+2)$ and $y = 1$ when $x = 0$

Selected Answers:

5. $y = \sqrt{x^2 + 3}$; all real numbers	6. $y = \sqrt{25 - x^2}$; valid on the interval $(-5, 5)$
7. $y = x$; valid on the interval $(0, \infty)$	8. $y = 3e^{x^2}$, valid for all real numbers
9. $y = 6e^{\frac{x^2}{2}+2x} - 5$, valid for all real numbers	

1. The graph of several solutions to the differential equation $\frac{dy}{dx} = \frac{x^2}{y^2}$ is shown. Solve the equation, then find the particular solution that satisfy the initial conditions (a) $y(0) = 2$, (b) $y(0) = -2$, and (c) $y(0) = 0$.



2. Find the general and particular solutions to the separable differential equation $\frac{dy}{dx} = x^2 y$ given the initial conditions (a) $f(0) = 1$ and (b) $f(0) = -2$.

3. Let f be a function with $f(1) = 4$ such that for all points (x, y) on the graph of f, the slope is given by $\frac{dy}{dx} = \frac{3x^2+1}{2y}$.

(a) Find the slope of the graph of f at the point where $x = 1$.

(b) Write an equation of the line tangent to the graph of f at $x = 1$ and use it to approximate $f(1.2)$.

(c) Find $f(x)$ by solving the separable differential equation $\frac{dy}{dx} = \frac{3x^2+1}{2y}$ with the initial condition $f(1) = 4$.

(d) Use your solution from part (c) to find the exact value of $f(1.2)$.

4. Consider the differential equation $\frac{dy}{dx} = 6 - 2y$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 4$.

(a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 0$. Use the tangent line to approximate $f(0.6)$.

(b) Find $\frac{d^2y}{dx^2}$. Is the approximation found in part (a) an overestimate or underestimate of the actual value of $f(0.6)$. Justify your answer.

(c) Find $y = f(x)$, the particular solution to the differential equation with the initial condition $f(0) = 4$.

(d) For the particular solution $y = f(x)$ found in part (c), find $\lim_{x \to \infty} f(x)$.

1. If $\frac{dN}{dt} = \frac{1}{5}N$, and $N(0) = 500$, then $N(t) =$

2. The rate of increase of a population is given by $\frac{dN}{dt} = \frac{1}{2}N$. If the initial population is 1000, then find the equation that will give the population at any time *t*.

3. The rate of mass decay for a certain radioactive substance is given by $\frac{dM}{dt} = -\frac{M}{50}$, *M* is in grams and *t* is in years. If the initial mass of the substance is 250 grams, what mass will remain when $t = 100\ln 2$ years?

4. If $f(t) = \int_0^{t^2} \frac{1}{1+x^2}\,dx$, find $f'(t)$.

5. Find the maximum value of the function $f(x) = x^4 - 4x^3 + 6$ on the interval $[1,4]$.

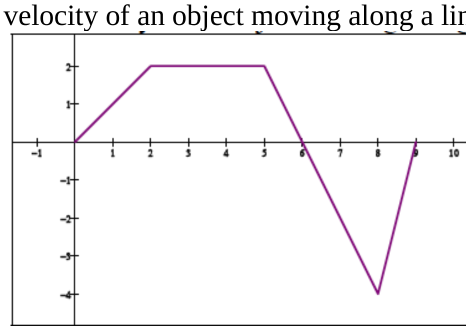
6. The rate of growth of a certain population equals one tenth of the present population. If the initial population is 5000, what will the population be in 20 years?

7. A differentiable function has values shown in this table:

x	2.0	2.2	2.4	2.6	2.8	3.0
$f(x)$	1.39	1.73	2.10	2.48	2.88	3.30

Estimate $f'(2.1)$ A) 0.34 B) 0.59 C) 1.56 D) 1.70 E) 1.91

For Questions 8-10, the graph shows the velocity of an object moving along a line, for $0 \le t \le 9$.



8. At $t = 8$, the object was at position $x = 10$. At $t = 5$, what is the object's position?

9. When was the object farthest from the starting point? (Hint: find its position at critical points and endpoints)

10. On what interval is the object moving to the left?

The table shown is for Questions 11 and 12. The differentiable functions *f* and *g* have the values shown.

x	***f***	***f '***	**g**	**g '**
1	2	1/2	–3	5
2	3	1	0	4
3	4	2	2	3
4	6	4	3	1/2

11. Find the average rate of change of the function *f* on $[1,4]$.

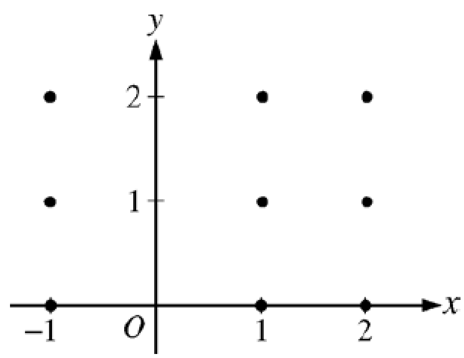
12. If $h(x) = g(f(x))$, find $h'(3)$.

2008 #5

Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

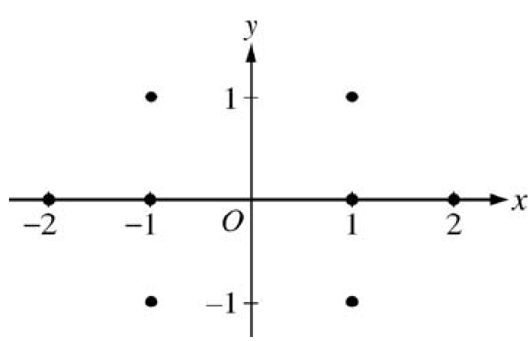
(c) For the particular solution $y = f(x)$ described in part (b), find $\lim_{x \to \infty} f(x)$.

2006 #5

Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.

(Note: Use the axes provided in the pink exam booklet.)

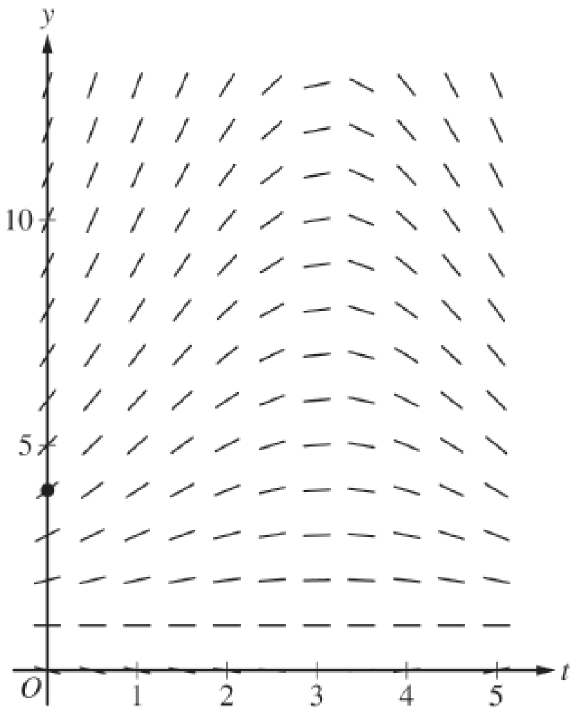


(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

2024 #3

3. The depth of seawater at a location can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$, where $H(t)$ is measured in feet and t is measured in hours after noon ($t = 0$). It is known that $H(0) = 4$.

(a) A portion of the slope field for the differential equation is provided. Sketch the solution curve, $y = H(t)$, through the point $(0, 4)$.



(b) For $0 < t < 5$, it can be shown that $H(t) > 1$. Find the value of t, for $0 < t < 5$, at which H has a critical point. Determine whether the critical point corresponds to a relative minimum, a relative maximum, or neither a relative minimum nor a relative maximum of the depth of seawater at the location. Justify your answer.

(c) Use separation of variables to find $y = H(t)$, the particular solution to the differential equation $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$ with initial condition $H(0) = 4$.

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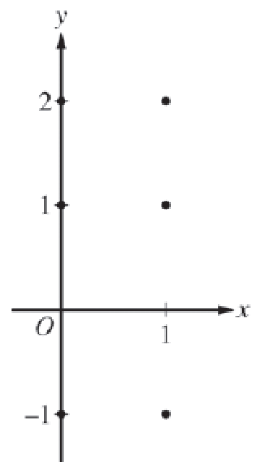
4. Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



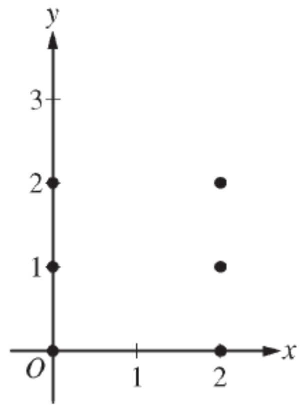
(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

(c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.

(d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.

4. Consider the differential equation $\frac{dy}{dx} = \frac{y^2}{x-1}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



(b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(2) = 3$. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 2$. Use your equation to approximate $f(2.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 3$.